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Regional Vector Trends in Time Series

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Abstract

Temperature of a region can be treated as a vector time series. In this article, we have treated temperature of Marathwada of Maharastra state as a vector $\bar{t} = (X_1, X_2, \ldots X_5)$. Where X_1 = temperature at Aurangabad, X_2 = temperature at Parbhani, X_3 = temperature at Osmanabad, X_4 = temperature at Beed and X_5 = temperature at Nanded. Thus, we get a vector time series, $\bar{T} = (t_{ij})$, i = 1, 2 , … n years, j = 1,2 ... 5 districts(districts having five temperature stations). This opens up very interesting questions. How are the properties of T related to component time series?.

A preliminary discussion of properties of vector time series and possible testing methodology for trend property precedes the actual application to regional temperature data.

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Introduction

Vector time series can occur naturally in real life. For example, if we consider the temperature over a region, where temperature is recorded over a cluster of recording stations, we get a vector temperature time series. To what extent the properties of component time series determine the properties of the regional vector time series is worth looking into.

In what follows are first discussed in relation to, few properties of vector time series and then tried to compute the same for the regional annual 33 years temperature record of Marathwada region by using data from 1970 to 2002.

Basic Concepts

Basic definitions and few properties of vector stationary time series are given in this section.

Definition 2.1: A Random Vector

A random vector, $\bar{X} = (X_1, X_2, \ldots X_K)$ is a single valued function whose domain is Ω, whose range is in Euclidean n-space R^n and which is B-measurable, i.e. for every subset R C R^n {ω Є Ω | X_1(ω) … X_K(ω) Є R}Є B. A random vector will also be called an K- dimensional random variable or a vector random variable.

If $X_1, X_2 \ldots X_K$ are k random variables and $\bar{X} = (X_1, X_2 \ldots X_K)$ is a random vector , [15].

Definition 2.2: A Vector Time Series

Let (Ω, C, P) be a probability space; with Ω sample space; C = σ(Ω). Let T be an index set and N = {1, 2 ... k}. A real valued vector time series is a real valued function $X_{it}(\omega)$, i = 1,2…k defined on N ×T ×Ω such that for each fixed t Є T, i Є N, $X_{it}(\omega)$ is a random variable on (Ω, C, P).

A vector time series can be considered as a collection {X_{it}: t Є T }, i = 1,2 ... k of random variables [16].

Definition 2.3: Stationary Vector Time Series

A process whose probability structure does not change with time is called stationary. Broadly speaking a vector time series is said to be stationary, if there is no systematic change in mean i.e. no trend. There is no systematic change in variance.

Let $\bar{X} = (x_1, x_2 \ldots x_n)$ be realizations of random variables $(X_1, X_2 \ldots X_K)$.

Definition 2.4: Strictly Stationary Vector Time Series

A vector time series is called strictly stationary, if their joint distribution function satisfy

$$F_{X_{1t} X_{2t} \dots X_{kt}}(\bar{X}) = F_{X_{1t+h} X_{2t+h} \dots X_{kt+h}}(\bar{X}) \quad \dots (1)$$

Where, the equality must hold for all possible sets of indices (it) and (it + h) in the index set. Further the joint distribution depends only on the distance h between the elements in the index set and not on their actual values.

Main Results

Theorm 3.1: If $\{ X_{it} : t \in T \}$, i =1, 2,…k is strictly vector time series with $E\{X_{it}\} < \alpha$ and

$E\{X_{it} - \mu\} < \beta$ then,

$$E\{X_{it}\} = E\{X_{it+h}\} \text{, for all it, h}$$

and $E[\{X_{it} - \mu_i\}\{X_{jt} - \mu_j\}] = E[\{X_{it+h} - \mu_i\}\{X_{jt+h} - \mu_j\}]$, for all it, h } … (2)

Proof : Proof follows from definition (2.4).

In usual cases above equation (2) is used to determine that a vector time series is stationary.

Definition 3.1: Weakly stationary vector time series

A vector time series is called weakly stationary if

1. The expected value of X_{it} is a constant for all it.
2. The covariance matrix of $(X_{1t}, X_{2t}, \dots X_{kt})$ is same as covariance matrix of $(X_{1t+h}, X_{2t+h} \dots X_{kt+h})$.

A look in the covariance matrix $(X_{1t}\ X_{2t} \dots X_{kt})$ would show that diagonal terms would contain terms covariance (X_{it}, X_{it}) which are essentially variances and off diagonal terms would contains terms like covariance (X_{it}, X_{jt}).Hence, the definitions to follow assume importance . Since these involve elements from the same set $\{X_{it}\}$, the variances and co- variances are called auto-variances and auto-co variances .

Definition 3.2: Auto-covariance function: The covariance between $\{X_{it}\}$and $\{X_{it+h}\}$ separated by h time unit is called auto-covariance at lag h and is denoted by $\Gamma_{ij}(h)$.

$$\Gamma_{ij}(h) = \text{cov}(X_{it}, X_{jt+h}) = E\{X_{it} - \mu_i\}\{X_{jt+h} - \mu_j\} \quad \dots (3)$$

The matrix $\Gamma\ \bar{h} = \Gamma_{ij}(h)$ is called the auto covariance matrix function.

Definition 3.3: The Auto Correlation Function

The correlation between observation which are separated by h time unit is called auto-correlation at lag h. It is given by

$$P_{ij}(h) = \frac{E\{X_{it} - \mu_i\}\{X_{jt+h} - \mu_j\}}{[E\{X_{it} - \mu_i\}^2 E\{X_{jt+h} - \mu_j\}^2]^{1/2}} \quad \dots (4)$$

$$= \frac{\Gamma_{ij}(h)}{[\Gamma_{ii}(h)\ \ \Gamma_{jj}(h)]^{1/2}}$$

Where, μ_i is the mean of i^{th} component time series.

Remark 3.1

For a vector stationary time series the variance at time (it + h) is same as that at time it. Thus, the auto correlation at lag h is

$$P_{ij}(h) = \frac{\Gamma_{ij}(h)}{\Gamma_{ii}(0)} \quad \dots (5)$$

Remark 3.2

For h = 0, we get $\rho_{ij}(0) = 1$.

For application attempts have been made to establish that temprature at certain districts of Marathwada satisfy equation (1) and (5).

Definition 3.4: Positive Semi-Definite

A function f(x) defined for $x \in X$ is said to be positive semi-definite if it satisfies

$$\sum_{J=i}^{n} \sum_{k=1}^{n} a_k^T f(t_j - t_k)\, a_j \geq 0,$$

For any set of real vectors $(a_1, a_2, \dots a_n)$ and any set of indices $(t_1, t_2 \dots t_n) \in T$ such that $(t_j - t_k) \in X$.

Theorem 3.2

The covariance function of vector stationary time series $\{X_{it} : t \in T\}$ is positive semi-definite function in that

$$\sum_{J=1}^{n} \sum_{k=1}^{n} a_k^T \Gamma(t_j - t_k)\, a_j \geq 0,$$

For any set of real vectors $(a_1, a_2, \dots a_n)$ and any set of indices $(t_1, t_2 \dots t_n) \in T$.

Proof: The result can be obtained by evaluating the variance of

$$X = \sum_{j=1}^{n} a_j^T X_{tj}.$$

For this without loss of generality $E(X_{tj}) = 0$. It shows that the variance of a random variable is non-negative i.e. $V(X) \geq 0$.

$$V(X) = V(\Sigma\ a_j^T X_{tj}) \geq 0$$

$$= E\left(\sum_{j=1}^{n} a_j^T X_{tj}\right)\left(\sum_{k=1}^{n} a_j^T X_{tj}\right)^T \geq 0,$$

$$= \sum_{J=1}^{n} \sum_{k=1}^{n} a_j^T a_k E\{X_{tj} X_{tk}\} \geq 0,$$

$$= \sum_{J=1}^{n} \sum_{k=1}^{n} a_k^T \Gamma(t_j - t_k)\, a_j \geq 0 \quad \dots (6)$$

Hence proved.

Theorem 3.3: **$|\rho_{12}(h)| \leq 1$.**

Proof: If we set n = 2, in the equation (6) to obtain,

$$\sum_{i=1}^{n} \sum_{j=1}^{n} a_j^T \Gamma(t_i - t_j)\, a_i = a_1^2\, \Gamma_{11}(0) + a_2^2\, \Gamma_{22}(0) + 2a_1 a_2^T \Gamma(t_1 - t_2) \geq 0.$$

$a_1^2\, \Gamma_{11}(0) + a_2^2\, \Gamma_{22}(0) \geq -2a_1 a_2^T\, \Gamma_{12}(t_1 - t_2)$, since $\Gamma_{11}(0) = \Gamma_{22}(0)$

$$1/2(a_1^2 + a_2^2) \geq \frac{-a_1 a_2^T\, \Gamma_{12}(t_1 - t_2)}{\Gamma_{11}(0)}$$

Now , let $a_1 = a_2 = 1$ and $t_1 - t_2 = h$,

$$1 \geq \frac{-\Gamma_{12}(h)}{\Gamma_{11}(0)} = -\rho_{12}(h) \quad \ldots (7)$$

Similarly, $-a_1 = a_2 = 1$; it shows that

$$P_{12}(h) \leq 1 \quad \ldots (8)$$

From (7) and (8) we get

$$|\rho_{12}(h)| \leq 1.$$

Hence proved.

Theorem 3.4

The auto covariance matrix of vector stationary time series is an even function of h. i.e., $\Gamma_{ij}(h) = \Gamma_{ij}(-h)^T$.

Proof: Here,

$$Cov(X_i, Y_{i+1}) = \{\Sigma X_i Y_{i+1} - 1/n \Sigma X_i \Sigma Y_i\}/n,$$

If X_i, Y_i are different series.

$$Cov(X_i, Y_{i+1}) \neq Cov(X_i, Y_{i-1})$$

i.e. $\{\Sigma X_i Y_{i+1} - 1/n\Sigma X_i \Sigma Y_i\}/n \neq \{\Sigma X_i Y_{i-1} - 1/n \Sigma X_i \Sigma Y_{i+1}\}/n$

$\therefore X_1 Y_2 \neq X_2 Y_1$

When X_i, Y_i are identical series

$$\Gamma(1) = \Gamma(-1),$$

Otherewise not true.

Hence, $\Gamma_{ij}(h) = \Gamma_{ij}(-h)^T$ proved.

Theorem 3.5: Let X_{it}'s be independently and identically distributed with $E(X_{it}) = \mu_i$ and $var(X_{it}) = \sigma_i^2$ then

$$\Gamma_{ij}(t, k) = E(X_{it}, X_{jk}) = \sigma_i^2, \quad t = k$$
$$= 0, \quad t \neq k$$

This process is stationary in the strict sense.

Testing Procedure

Using the model for table-5.1B

$X_i = (\beta_0)_i + (\beta_1)_i t_i + \epsilon_i$, i =1, 2, … 5 … (9)

Where (i) X_i are annual temperature series $X_i(t)$, i = 1, 2 … 5 for five districts.

(ii) t_i are the time (in years) variable.

(iii) ϵ_i, are a random error term normally distributed as mean zero and variance σ^2, i.e $\epsilon_i \sim N(0, \sigma^2)$.

Temperature X_i (degree centigrade) are the dependent variables and time t_i (in years) are independent variables.

Using the model for table-5.2C and 5.3C

$\Upsilon_{ij}(h) = (\beta_0)_{ij} + (\beta_1)_{ij}\, h + \epsilon_{ij}$, i =1, 2, … 5 ; j = 1, 2, … 5 … (10)

Where (i) $\Upsilon_{ij}(h)$ are auto-variance values for individual series and auto-covariance matrices for vector time series.

(ii) h are the lag values of variable.

(iii) ϵ_i are a random error term normally distributed as mean zero and variance σ^2, i.e. $\epsilon_i \sim N(0, \sigma^2)$. $\Upsilon_{ij}(h)$ are the dependent variables and h are independent variables.

$\Upsilon_{ij}(h)$ are the dependent variables and h are independent variables.

Defining $\Upsilon_{ij}(h) = cov(X_i, X_{j+h})$, (i = 1, 2 … 5; j =1, 2 … 5) were computed for various values of h by using **MS-Excel**. Since the total series constituted of 33 values at least 10 values were included in the computation. The relation between $\Upsilon_{ij}(h)$ and h were examined the model in (table-5.2C).

Defining the $\Gamma_{ij}(h)) = cov(X_i, X_{j+h})$, covariance matrix with a stationary time series for observations $\bar{X} = (x_1, x_2, \ldots x_n)$ realizations and ρ_{ij} (h) = correlation (X_i, X_{j+h}) correlation matrix with a stationary time series for observations $\bar{X} = (x_1, x_2, \ldots x_n)$ realizations , 21 such matrices corresponding to h = 0 to 20 , define one series of matrices each 5×5 , and hence 25 component series were computed. The relation between $\Gamma_{ij}(h)$ and h were examined the model in (table-A1 , APPENDIX-A) .

The method of testing intercept $(\beta_0)_{ij} = 0$ and regression coefficient $(\beta_1)_{ij} = 0$, [16]. Null hypothesis for test Statistic used to test and set up.

4.1: Inference concerning slope $(\beta_1)_{ij}$: For testing $H_0: (\beta_1)_{ij} = 0$ Vs $H_1: (\beta_1)_{ij} > 0$ for $\alpha = 0.05$ percent level using t distribution with degrees of freedom is equal to n – 2 were considered.

$$t_{n-2} = \beta_1 / s_{\beta 1}$$

From model, $\Gamma_{ij}(h) = (\beta_0)_{ij} + (\beta_1)_{ij}\, h + \epsilon_i$,

Table-5.3C was obtained by regressing values of $\Gamma_{ij}(h)$ against h, by using this , testing shows that , both the hypothesis $(\beta_0)_{ij} = 0$ and $(\beta_1)_{ij} = 0$ test is not positive. (Table-A1, APPENDIX-A) formed the input for table-5.3A .

Example of vector time series

Regional temperature data.

Here is a real instance of a vector time series. Temperature data of Marathawada region was obtained from five districts, namely Aurangabad, Parbhani, Beed, Osmanabad, and Nanded. The data were collected from *"Socio Economic Review and District Statistical Abstract",* Directorate of Economic and Statistics, Government of Maharastra, Bombay and *"Hand Book of Basic Statistics",* Maharastra State [**2, 3, 4**]. Hence we have five dimensional time series t_i , i = 1, 2, 3, 4, 5 corresponding to the districts Aurangabad, Parbhani, Beed, Osmanabad and Nanded respectively. Table 5.1A, shows the results of descriptive statistics and Table 5.1B, shows linear trend analysis. All the linear trends were found to be not significant.

Over the years many scientists have analyzed rainfall, temperature, humidity, agricultural area, production and productivity of Marathwada region of Maharastra state, [**1, 5, 6, 8, 9, 11, 13, 14, 15, 18**]. Most of them have treated the time series for each of the revenue districts as independent time series and tried to examine the stability or non-stability depending upon series. Most of the times non- stability has been concluded, and hence possibly any sort of different treatment was possibility never thought of. In this investigation we treat the series first and individual series and then as a vector time series shows that the vector time series are not stable.

Conclusion

For Individual Time Series

It was observed t values are therefore not significant for the four districts i.e. X_i does not depend on time t for four districts [5]. Similarly, $\Upsilon_{ij}(h)$ does not depend on h in four districts except Osmanabad districts to mean that there is trend in Osmanabad districts. The testing shows that, for the hypothesis $(\beta_1)_i = 0$, test is negative for time t and h, for four districts i.e. trends were not found in four districts i.e.

trends were not found in four districts except Osmanabad district.

Generally it is expected, temperature (annual) over a long period at any region to be stationary time series. These results does not conform with the series in Osmanabad district.

b) For vector time series

It is necessary to test association between Γ_{ij} (h)) and h. Conclude that a vector time series was having trend, The association between Γ_{ij} (h)) and h fails in Osmanabad–0.526*individually and Osmanabad / Aurangabad –0.526*, Osmanabad / Parbhani –0.526* Osmanabad / Beed–0.526* and Parbhani / Nanded –0.526* in combinations are seems to be causing responsible for having trend nature of the series (shown in table-5.3B) it is concluded that a vector time series found trend[16].

Analysis: Temperature

The strategy of analyzing first individual time series as scalar series and then treating the vector series as the regional time series has been adapted here for temperature.

Temperature Time Series Treated As Scalar Time Series

Table 5.1A contains the results for scalar series approach.

Table-5.1A: Elementary statistics of *temperature* data (in degree centigrade C^0) of Marathwada region for 33 years (1970-2002).

Cities:	Aurangabad	Parbhani	Osmanabad	Beed	Nanded
Mean:	40.46	42.78	40.73	41.27	43.04
S.D.:	2.00	1.71	1.85	2.30	1.39
C.V.:	3.49	3.06	3.34	3.68	2.74

Table-5.1B:Test of significance for $\beta_1 = 0$ the model : $X_i(t) = (\beta_0)_i + (\beta_1)_i t + \epsilon_i$, i =1, 2,…5

District	Coefficients		Standard Error	t Stat	Significance
Aurangabad	β_0	40.80	0.73	55.87	S
	β_1	-0.02	0.04	-0.52	NS
Parbhani	β_0	43.26	0.62	69.53	S
	β_1	-0.03	0.03	-0.87	NS
Osmanabad	β_0	39.34	0.62	63.91	S
	β_1	0.08	0.03	**2.59***	**S**
Beed	β_0	41.44	0.85	49.00	S
	β_1	-0.01	0.04	-0.23	NS
Nanded	β_0	43.38	0.51	85.68	S
	β_1	-0.02	0.03	-0.77	NS

*t =2.04 is the critical value for 31 d f at 5% L. S. * shows the significant value*

A look at the table 5.1A shows that all of them have similar values of CV. Which indicates that their dispersion is almost identical. Trends were found to be not significant in 4 districts but ***significant*** in ***Osmanabad*** district only. A simple look at the mean values shows that a classification as

C1 = {Aurangabad, Osmanabad}

C2 = {Nanded, Parbhani, Beed} could be quite feasible.

Trend: Absence of linear trend, with reasonably low CV values can be taken as evidence of series being stationary series individually in four districts, but in ***Osmanabad*** districts trends were found.

Table-5.2A: **Auto variances: Individual column treated as ordinary time series for lag values (h = 0, 1, 2 … 20) about *temperature* data.**

lag h	Aurangabad	Parbhani	Osmanabad	Beed	Nanded
0	4.0	2.9	3.4	5.3	1.9
1	0.1	0.2	2.4	1.0	0.8
2	-0.3	-0.3	1.6	-0.2	-0.1
3	-0.4	0.5	1.1	0.1	-0.3
4	0.7	0.3	0.8	-0.5	-0.2
5	-1.0	-0.4	0.2	-1.5	-0.5
6	-0.6	-0.7	-0.4	0.2	-0.2
7	0.2	0.6	-0.7	0.2	0.6
8	0.2	-0.1	-0.1	-0.4	0.6
9	-0.3	-0.2	0.0	-0.6	-0.6
10	0.0	-1.1	-0.2	-0.5	-0.8
11	0.8	-0.1	-0.8	-0.4	-0.5
12	-0.1	0.2	-0.6	1.1	-0.5
13	0.2	-1.2	-0.2	-1.5	-0.8
14	-0.6	-1.1	-0.2	-1.0	-0.2
15	0.3	0.4	0.2	-0.5	0.6
16	-1.9	0.7	0.5	-0.1	0.5
17	0.1	-0.5	0.7	-1.7	-0.1
18	0.2	0.0	0.8	1.8	0.0

19	0.2	1.1	0.2	1.7	-0.1
20	-2.2	0.3	-0.4	-1.3	-0.4

Table-5.2B: Correlation coefficient between h and Auto covariance is:

Districts	Aurangabad	Parbhani	Osmanabad	Beed	Nanded
Corr. Coefficient	-0.426	-0.207	**-0.526***	-0.292	-0.333

*Correlation coefficient r = 0.433 is the critical value for 19 d f at 5% L. S. * shows the significant value.*

Correlation's between Υ_{ij} (h) and h were found *significant* in *Osmanabad* district only showing that the time series can be reasonably assumed to be not stationary. The coefficient is significant, with negative value showing that Osmanabad has been experiencing significantly declining temperature over the past years.

Table-5.2C : Test of significance for $\beta_1 = 0$ the model : $\Upsilon_{ij}(h) = (\beta_0)_{ij} + (\beta_1)_{ij} h + \varepsilon_{ij}$, (i = 1 , 2 ,…5: j = 1 , 2…5)

District	Coefficients		Standard Error	t Stat	Significance
Aurangabad	β_0	0.78	0.46	1.70	NS
	β_1	-0.08	0.04	-2.05	NS
Parbhani	β_0	0.38	0.38	1.00	NS
	β_1	-0.03	0.03	-0.92	NS
Osmanabad	β_0	1.28	0.38	3.35	S
	β_1	-0.09	0.03	-2.70*	S
Beed	β_0	0.79	0.64	1.23	NS
	β_1	-0.07	0.05	-1.33	NS
Nanded	β_0	0.34	0.27	1.27	NS
	β_1	-0.04	0.02	-1.54	NS

*t =2.1 is the critical value for 19 d f at 5% L. S. * shows the significant value*

Temperature time series of five districts treated as a single vector time series

Treating the series together, one may look at them as a single vector time series for the whole region, with each vector T = (t_1 , t_2 , t_3 , t_4 , t_5) having five components. Where t_i is the temperature for the i th districts.

Now for each lag value h we have a 5x5 matrix Γ_{ij} (h), of auto and cross covariance values. These values, which constitute of 21 such matrices is reported in Table-5.3 Appendix-A.

Observe that,

a. the matrix Υ_{ij} (h) for h = 0 is symmetric, and for h > 0 they are not symmetric. This is expected.

b. We have a series of 5 × 5 matrices Υ_{ij} (h), h = 0,1,…, 20 , and now onwards we are interested in behavior of this matrix series.

c. Out of the 25 components series, 5 series showed significant (coefficients)

intercepts, and slope in table 5.3B Osmanabad–0.526* individually and Osmanabad/Aurangabad–0.526*, Osmanabad/Parbhani–0.526* Osmanabad/ Beed–0.526* and Parbhani/Nanded–0.526* in combinations are seems to be causing responsible for non-stationary nature of the series. That is both hypothesis $\beta_0 = 0$ and $\beta_1 = 0$ could be rejected. Stated in matrix terms the model (matrix equation in 5 × 5 matrices)

$\Gamma_{ij}(h) = \beta_{ij}(0) + \beta_{ij}(1)h + e_{ij}(h)$, (i = 1,2 … 5 ; j = 1,2 … 5)

with hypothesis $\beta_0 = 0$ and $\beta_1 = 0$ was not validated.

Hence we can consider the vector time series to be *not stationary*. Thus we have a situation where the vector time series is not stationary, so the individual time series are stationary except Osmanabad districts.

Table-5.3A: Cov.(h, Γ_{ij}(h)) Matrix values about *temperature* data.

District	Aurangabad	Parbhani	Osmanabad	Beed	Nanded
Aurangabad	-2.97	-0.23	1.21	-2.23	-1.71
Parbhani	-1.19	-1.10	0.42	-1.27	-1.41
Osmanabad	2.53	3.65	-3.23	3.58	2.01
Beed	-2.59	0.06	0.73	-2.67	-2.24
Nanded	-0.63	0.06	-0.08	-0.49	-1.30

Table-5.3B: ρ_{ij}(h) = Correlation(h, Γ_{ij}(h)) Matrix values about *temperature* data.

District	Aurangabad	Parbhani	Osmanabad	Beed	Nanded
Aurangabad	-0.426	-0.044	0.222	-0.304	-0.353
Parbhani	-0.431	-0.207	0.165	-0.307	**-0.444***
Osmanabad	**0.593***	**0.628***	**-0.526***	**0.521***	0.409
Beed	-0.415	0.008	0.123	-0.292	-0.402
Nanded	-0.249	0.013	-0.028	-0.126	-0.333

*Correlation coefficient r = 0.433 is the critical value for 19 d f at 5% L. S. * shows the significant value.*

Table-5.3C: Test of significance for β_1 =0, the model $\gamma ij(h) = \beta ij(0) + \beta ij(1)h + eij(h)$, (i = 1,2…5 ; j = 1,2 …5)

District/District	coefficient		Stand. error	t-stat	Significance
Aurangabad/Aurangabad	β_0	0.78	0.46	1.70	NS
	β_1	-0.08	0.04	-2.05	NS
Aurangabad/Parbhani	β_0	0.18	0.37	0.48	NS
	β_1	-0.01	0.03	-0.19	NS
Aurangabad/Osmanabad	β_0	-0.16	0.39	-0.41	NS
	β_1	0.03	0.03	0.99	NS
Aurangabad/Beed	β_0	0.71	0.51	1.39	NS
	β_1	-0.06	0.04	-1.39	NS
Aurangabad/Nanded	β_0	0.48	0.33	1.45	NS
	β_1	-0.05	0.03	-1.65	NS
Parbhani/Aurangabad	β_0	0.29	0.18	1.61	NS
	β_1	-0.03	0.02	-2.08	NS
Parbhani/Parbhani	β_0	0.38	0.38	1.00	NS
	β_1	-0.03	0.03	-0.92	NS
Parbhani/Osmanabad	β_0	0.05	0.19	0.27	NS
	β_1	0.01	0.02	0.73	NS
Parbhani/Beed	β_0	0.36	0.29	1.26	NS
	β_1	-0.03	0.02	-1.41	NS
Parbhani/Nanded	β_0	0.36	0.21	1.75	NS
	β_1	-0.04	0.02	-2.16*	S
Osmanabad/Aurangabad	β_0	-0.78	0.25	-3.11	S
	β_1	0.07	0.02	3.21*	S
Osmanabad/Parbhani	β_0	-1.04	0.33	-3.16	S
	β_1	0.10	0.03	3.52*	S
Osmanabad/Osmanabad	β_0	1.28	0.38	3.35	S
	β_1	-0.09	0.03	-2.70*	S
Osmanabad/Beed	β_0	-1.06	0.43	-2.47	S
	β_1	0.10	0.04	2.66*	S
Osmanabad/Nanded	β_0	-0.68	0.33	-2.07	NS
	β_1	0.05	0.03	1.95	NS
Beed/Aurangabad	β_0	0.58	0.42	1.40	NS
	β_1	-0.07	0.04	-1.99	NS
Beed/Parbhani	β_0	0.04	0.50	0.09	NS
	β_1	0.00	0.04	0.04	NS
Beed/Osmanabad	β_0	0.20	0.43	0.46	NS
	β_1	0.02	0.04	0.54	NS
Beed/Beed	β_0	0.79	0.64	1.23	NS
	β_1	-0.07	0.05	-1.33	NS
Beed/Nanded	β_0	0.56	0.37	1.49	NS
	β_1	-0.06	0.03	-1.91	NS
Nanded/Aurangabad	β_0	0.14	0.18	0.81	NS
	β_1	-0.02	0.02	-1.12	NS
Nanded/Parbhani	β_0	0.04	0.31	0.12	NS
	β_1	0.00	0.03	0.06	NS
Nanded/Osmanabad	β_0	0.22	0.20	1.13	NS
	β_1	0.00	0.02	-0.12	NS
Nanded/Beed	β_0	0.15	0.28	0.54	NS
	β_1	-0.01	0.02	-0.55	NS
Nanded/Nanded	β_0	0.34	0.27	1.27	NS
	β_1	-0.04	0.02	-1.54	NS

*t =2.1 is the critical value for 19 d f at 5% L. S. * shows the significant value*

Thus we have a situation where when treated as individual series *Osmanabad* is not stationary and when treated as vector time series the whole of the vector time series is not stationary. This confirms one of the properties of vector time series that

If one of the component series is non stationary, the vector series as a whole is also non stationary.

When we think of the maximum temperature of whole of the region, Osmanabad individually and Osmanabad/Parbhani, Osmanabad/Beed, Osmanabad/Aurangabad, and Parbhnai/Nanded in combinations seem to be causing the variations responsible for the nonstationary nature of the series.

Regional View of the Temperature Aspects

A regional view of the temperature aspects helps us in classifying the region into subgroups of similar temperature characteristics. From results it is clear that Aurangabad, Parbhani, Beed and Nanded are stationary in temperature time series however and Osmanabad are not so. When the vector time series is considered the region showed non-stationary behavior. This suggests that, the districts can be clustered into sub-regions which we show us homogeneous behavior. With this intention the simple analysis of variance (ANOVA) was performed. The districts are treated as a single factor and years as replicates.

5. 4A: ANOVA-table: For temperature series

Source of variations	SS	d. f.	MS	F	P-value	F crit
Between group	184.9	4.0	46.2	**12.8***	0.0	2.4
With in group	579.8	160.0	3.6			
Total	764.7	164.0				

CD = 1.2

We need only to inspect the 5 means to see any pair which differs from the CD value.

Nanded	=	43.04	=	A1		
Parbhani	=	42.78	=	A2	⇒	A1 - A2 = 0.2
Beed	=	41.26	=	A3	⇒	A2 - A3 = **1. 5***
Osmanabad	=	40.73	=	A4	⇒	A3 - A4 = 0.6
Aurangabad	=	40.46	=	A5	⇒	A4-A5 = 0.2

Since one result, in a difference, is greater than our CD; these two districts Parbhani and Beed differ significantly i.e. they are not at par. So we can state that these two districts are not at state value. It means that they are below the face value or may be above the face value. If we consider in terms of temperature, the Nanded – CD = 43.04 – 1.2 = 41.84. It is seen that Nanded and Parbhani has greater mean values with the difference value of 41.84. So Nanded and Parbhani are at par. It means that they are at face value

Here from the ANOVA table, if F is significant; it is seen that the five means are not equal. Here we have five comparisons of interest. For this, we calculate CD(Critical Difference) as above. We have also arranged the means in the descending order of five districts and subtracted from CD. Therefore it is clear that Nanded and Parbhani are at par for temperature. Again we have taken corresponding difference among their respective means for the description of each temperature factor. In comparison, the temperature in Parbhani and Beed is not at par.

Summary of temperature Result

Table 5.4B: Temperature Time Series: Summary

Sr. No.	Factors	Scalar time series		Vector time series
		Stationary	Not stationary	
2	temperature	Aurangabad, Parbhani, Beed, Nanded	**Osmanabad**	Not stationary at $\rho_{25}(h)$, $\rho_{31}(h)$, $\rho_{32}(h)$, $\rho_{33}(h)$, $\rho_{34}(h)$

I) Time series for Aurangabad, Parbhani, Beed and Nanded are stationary i.e. not having trend.
II) Time series for ***Osmanabad*** has been unstable in temperature.
III) In weather-wise classification it is seen that ***Nanded*** and ***Parbhani*** are at par in temperature (see figure 5.1).

Districts → / Factors ↓	Nanded	Parbhani	Beed	Osmanabad	Aurangabad
Temperature	*---------------	--------------*	*	*	*

Figure-5.1

Appendix-A

Table-5.3: Auto variance and Auto covariance matrices: Vector time series. (i = 1, 2 … 5 :Districts; j =1, 2 … 5:Districts; h =1, 2 … 20: Lag values; Υ_{ij} = Cov.(i, j) about *temperature* data.

Lag h		j =1	j = 2	j = 3	j = 4	j = 5
0	I=1	4.0	1.1	0.1	3.3	1.3
	i=2	1.1	2.9	0.3	1.3	1.1
	i=3	0.1	0.3	3.4	0.7	0.7
	i=4	3.3	1.3	0.7	5.3	1.7
	i=5	1.3	1.1	0.7	1.7	1.9
1	i=1	0.1	0.7	-0.6	1.1	0.6
	i=2	0.5	0.2	0.0	0.9	0.2
	i=3	0.2	0.3	2.4	-0.1	0.6
	i=4	0.7	1.4	0.0	1.0	1.1
	i=5	0.3	0.7	0.6	1.0	0.8
2	i=1	-0.3	0.1	-0.2	-0.3	-0.1
	i=2	0.4	-0.3	-0.1	0.3	-0.5
	i=3	-0.5	-0.2	1.6	0.1	-0.2
	i=4	-0.4	0.4	0.1	-0.2	-0.1
	i=5	0.3	0.4	0.1	0.1	-0.1
3	i=1	-0.4	-0.1	0.1	0.2	-0.2
	I=2	0.1	0.5	-0.1	0.7	0.8
	I=3	-0.5	-0.3	1.1	0.0	-0.3
	I=4	0.5	-0.3	0.1	0.1	0.0
	I=5	0.0	0.5	0.0	0.4	-0.3
4	i=1	0.7	0.6	-0.1	-0.8	0.3
	i=2	0.4	0.3	0.3	0.7	0.7
	i=3	-0.8	-0.6	0.8	-0.4	-0.5
	i=4	-0.3	0.4	-0.1	-0.5	0.2
	i=5	0.0	0.5	0.2	0.0	-0.2
5	i=1	-1.0	-0.3	-1.0	-0.8	-0.5
	i=2	-0.2	-0.4	0.1	-0.2	0.2
	i=3	-0.4	-1.0	0.2	-1.5	-0.9
	i=4	-1.0	-0.7	-0.8	-1.5	-0.1
	i=5	-0.7	-1.1	0.1	-0.6	-0.5
6	i=1	-0.6	-1.5	-0.2	0.0	-0.4
	i=2	-0.4	-0.7	0.1	-0.8	-0.1
	i=3	-1.1	-2.1	-0.4	-1.3	-1.2
	i=4	-1.0	-1.7	0.5	0.2	-1.0
	i=5	-0.3	-1.6	0.0	-0.7	-0.2
7	i=1	0.2	0.6	0.4	1.0	0.8
	i=2	0.0	0.6	0.6	0.2	0.3
	i=3	-1.1	-1.7	-0.7	-1.8	-0.8
	i=4	0.4	-0.3	0.8	0.2	0.6
	i=5	-0.2	-0.3	0.4	0.1	0.6
8	i=1	0.2	1.1	0.7	0.5	1.1
	i=2	-0.1	-0.1	0.6	0.3	0.4
	i=3	-1.4	-0.7	-0.1	-1.5	-0.5
	i=4	-0.8	1.7	0.8	-0.4	0.5
	i=5	-0.3	0.4	0.3	-0.3	0.6
9	i=1	-0.3	-0.1	-0.2	-0.6	-0.5

	i=2	-0.8	-0.2	-0.2	-1.1	-0.6
	i=3	-0.3	-1.1	0.0	-1.7	-0.6
	i=4	-0.4	-0.9	0.4	-0.6	-0.3
	i=5	-0.7	-0.1	0.1	-1.0	-0.6
10	i=1	0.0	-0.4	-0.4	-0.2	0.0
	i=2	-0.3	-1.1	-0.3	-0.6	-0.8
	i=3	-0.5	-0.8	-0.2	-1.4	-1.3
	i=4	-0.5	-1.1	0.0	-0.5	-0.4
	i=5	0.0	-0.5	-0.2	-0.8	-0.8
11	i=1	0.8	0.9	-0.4	0.6	1.0
	i=2	0.3	-0.1	-0.6	0.2	-0.1
	i=3	-0.1	0.0	-0.8	-0.9	-0.9
	i=4	0.5	0.8	-0.5	-0.4	0.8
	i=5	0.0	0.0	-0.4	0.2	-0.5
12	I=1	-0.1	0.0	1.2	1.0	0.0
	i=2	-0.5	0.2	-0.2	-0.3	-0.1
	i=3	-0.4	0.2	-0.6	0.5	-1.0
	i=4	0.1	0.1	0.8	1.1	-0.7
	i=5	-0.3	-0.2	-0.3	-0.2	-0.5
13	i=1	0.2	-1.1	0.2	-1.9	-1.2
	i=2	-0.4	-1.2	0.0	-1.6	-0.9
	i=3	0.3	-0.2	-0.2	-0.2	-0.6
	i=4	0.4	-1.4	-0.2	-1.5	-1.4
	i=5	-0.3	-1.0	-0.5	-0.8	-0.8
14	i=1	-0.6	-0.9	0.7	-0.3	-1.1
	i=2	0.1	-1.1	0.2	-0.3	-0.4
	i=3	0.3	0.2	-0.2	-0.2	-0.4
	i=4	-1.4	-1.1	0.8	-1.0	-1.8
	i=5	0.3	-0.8	0.2	-0.2	-0.2
15	i=1	0.3	1.0	2.0	0.3	0.7
	i=2	0.2	0.4	1.0	0.1	0.4
	i=3	1.4	0.5	0.2	1.0	0.8
	i=4	0.1	0.7	2.3	-0.5	0.4
	i=5	0.3	0.5	0.8	0.0	0.6
16	i=1	-1.9	-0.9	2.0	-0.6	0.4
	i=2	-0.8	0.7	0.9	-0.5	0.0
	i=3	0.8	0.9	0.5	1.9	1.3
	i=4	-0.9	-1.8	2.7	-0.1	0.7
	i=5	-0.2	0.5	1.0	-0.2	0.5
17	i=1	0.1	-1.0	1.5	-1.9	-0.2
	i=2	0.0	-0.5	0.8	-0.3	-0.6
	i=3	0.1	1.2	0.7	1.1	1.1
	i=4	0.0	0.3	1.8	-1.7	0.7
	i=5	0.1	-0.1	1.0	0.1	-0.1
18	i=1	0.2	0.3	0.0	1.5	0.4
	i=2	0.3	0.0	0.4	0.7	-0.3
	i=3	0.4	1.0	0.8	1.0	0.2
	i=4	-0.3	0.4	0.5	1.8	0.4
	i=5	0.2	0.4	0.5	0.8	0.0
19	i=1	0.2	0.6	-0.7	1.4	0.2
	i=2	-0.1	1.1	0.0	0.6	0.3

	i=3	1.2	1.2	0.2	1.4	0.9
	i=4	0.4	0.5	-0.6	1.7	-0.6
	i=5	-0.1	1.3	0.1	0.7	-0.1
20	i=1	-2.2	1.8	-1.5	-1.5	-2.0
	i=2	-0.4	0.3	-0.4	0.1	-0.3
	i=3	0.3	1.7	-0.4	1.4	1.0
	i=4	-2.0	2.6	-1.7	-1.3	-2.0
	i=5	-0.2	0.4	-0.5	0.2	-0.4

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